Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Thursday 08 October 2020

Afternoon | Paper Reference **8FM0/23**

Further Mathematics
Advanced Subsidiary
Further Mathematics options
23: Further Statistics 1
(Part of options B, E, F and G)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. If a calculator is used instead of the tables, the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

1. A plumbing company receives call-outs during the working day at an average rate of 2.4 per hour.

(a) Find the probability that the company receives exactly 7 call-outs in a randomly selected 3-hour period of a working day.

(2)

The company has enough staff to respond to 28 call-outs in an 8-hour working day.

(b) Show that the probability that the company receives more than 28 call-outs in a randomly selected 8-hour working day is 0.022 to 3 decimal places.

(2)

In a random sample of 100 working days each of 8 hours,

(c) (i) find the expected number of days that the company receives more than 28 call-outs,

(1)

(ii) find the standard deviation of the number of days that the company receives more than 28 call-outs,

(2)

(iii) use a Poisson approximation to estimate the probability that the company receives more than 28 call-outs on at least 6 of these days.

(3)

a) Let X be the average rate of call-outs received per 3 hours.

$X \sim Po(2.4 \times 3)$

$X \sim Po(7.2)$

$P(X=7) = 0.1486$ (4dp) (using the calculator to save time in the exam)

b) Let Y be the average rate of call-outs received per 8 hours.

$Y \sim Po(2.4 \times 8)$

$Y \sim Po(19.2)$

$P(Y>28) = 1 - P(Y \leq 28) = 1 - 0.9780$

$= 0.0220$ (4dp)

ci) $E(z) = np = 100 \times 0.0220$

$= 2.20$ (3sf)

cii) $Var(z) = \sigma^2 = np(1-p)$

$\sigma = \sqrt{np(1-p)} = \sqrt{(100 \times 0.0220)(1-0.0220)}$

$= 1.47$ (3sf)

Question 1 continued

c iii) $Z \sim B(100, 0.0220)$

$Z \sim Po(100 \times 0.0220)$ $\quad Z \sim Po(np)$

$Z \sim Po(2.2)$

$P(Z \geqslant 6) = 1 - P(Z \leq 5) = 1 - 0.9751$

$= 0.0249$ (4dp)

(Total for Question 1 is 10 marks)

2. In an experiment, James flips a coin 3 times and records the number of heads. He carries out the experiment 100 times with his left hand and 100 times with his right hand.

	Number of heads				
	0	1	2	3	Total
Left hand	7	29	42	22	100
Right hand	13	35	36	16	100
Total	20	64	78	38	200

(a) Test, at the 5% level of significance, whether or not there is an association between the hand he flips the coin with and the number of heads.

You should state your hypotheses, the degrees of freedom and the critical value used for this test.

(7)

(b) Assuming the coin is unbiased, write down the distribution of the number of heads in 3 flips.

(1)

(c) Carry out a χ^2 test, at the 10% level of significance, to test whether or not the distribution you wrote down in part (b) is a suitable model for the number of heads obtained in the 200 trials of James' experiment.

You should state your hypotheses, the degrees of freedom and the critical value used for this test.

(7)

a) H_0 : There is no association between the hand and the number of heads.

H_1 : There is an association between the hand and the number of heads.

O_i	7	29	42	22	13	35	36	16
E_i	$\frac{20\times100}{200}$ $=10$	$\frac{64\times100}{200}$ $=32$	$\frac{78\times100}{200}$ $=39$	$\frac{38\times100}{200}$ $=19$	$\frac{20\times100}{200}$ $=10$	$\frac{64\times100}{200}$ $=32$	$\frac{78\times100}{200}$ $=39$	$\frac{38\times100}{200}$ $=19$
$\frac{(O_i-E_i)^2}{E_i}$	$\frac{(10-7)^2}{10}$ $=0.9$	$\frac{(32-29)^2}{32}$ $=0.281...$	0.230...	0.473...	0.9	0.281...	0.230...	0.473...

Question 2 continued

$$\sum \frac{(O_i - E_i)^2}{E_i} = 3.771\ldots$$

Degrees of freedom $= (v-1)(u-1)$
$= (4-1)(2-1)$
$= 3$

$\chi^2_3(0.05) = 7.815$ (Read from the table in your equation booklet)

As $3.771 < 7.815$
do not reject. There is insufficient evidence to suggest an association between the hand flipping the coin and the number of head.

b) Let X be the times the coin lands on heads in 3 tosses.
$X \sim B(3, 0.5)$

c) H_0: $X \sim B(3, 0.5)$ is a suitable model.
H_1: $X \sim B(3, 0.5)$ is not a suitable model.

No. of heads	O_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$
0	20	$200 \times P(X=0) = 25$	1
1	64	$200 \times P(X=1) = 75$	$1.61\dot{3}$
2	78	$200 \times P(X=2) = 75$	0.12
3	38	$200 \times P(X=3) = 25$	6.76

$$\sum \frac{(O_i - E_i)^2}{E_i} = 9.493$$

Degree of freedom $= 4 - 1 = 3$

$\chi^2_3(0.1) = 6.251$

As $9.493 > 6.251$
Reject H_0. Sufficient evidence to suggest that $B(3, 0.5)$ is not a suitable model.

(Total for Question 2 is 15 marks)

3. The probability distribution of the discrete random variable X is

$$P(X = x) = \begin{cases} \dfrac{k}{x} & \text{for } x = 1, 2 \text{ and } 3 \\ \dfrac{m}{2x} & \text{for } x = 6 \text{ and } 9 \\ 0 & \text{otherwise} \end{cases}$$

where k and m are positive constants.

Given that $E(X) = 3.8$, find $Var(X)$

(7)

$\Sigma p = 1 \quad \Rightarrow k + \frac{k}{2} + \frac{k}{3} + \frac{m}{12} + \frac{m}{18} = 1$

$\frac{11k}{6} + \frac{5m}{36} = 1 \quad \Rightarrow \quad 66k + 5m = 36 \quad - 1$

$\times 36$

$E(X) = 3.8 \quad \Rightarrow k + \frac{k}{2}(2) + \frac{k}{3}(3) + \frac{m}{12}(6) + \frac{m}{18}(9) = 3.8$

$3k + m = 3.8 \quad - 2$

$1 - 5 \times 2$

$66k + 5m - (15k + 5m) = 36 - 19$

$51k = 17$

$k = \frac{1}{3} \qquad m = \frac{14}{5}$

$E(X^2) = (1^2)k + (2^2)\frac{k}{2} + (3^2)\frac{k}{3} + (6^2)\frac{m}{12} + 9^2\left(\frac{m}{18}\right)$

$= 23$

$Var(X) = E(X^2) - (E(X))^2$

$= 23 - 3.8^2$

$= 8.56 \; (3sf)$

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 7 marks)

4. During the morning, the number of cyclists passing a particular point on a cycle path in a 10-minute interval travelling eastbound can be modelled by a Poisson distribution with mean 8

The number of cyclists passing the same point in a 10-minute interval travelling westbound can be modelled by a Poisson distribution with mean 3

(a) Suggest a model for the total number of cyclists passing the point on the cycle path in a 10-minute interval, stating a necessary assumption.

(2)

Given that exactly 12 cyclists pass the point in a 10-minute interval,

(b) find the probability that at least 11 are travelling eastbound.

(3)

After some roadworks were completed, the total number of cyclists passing the point in a randomly selected 20-minute interval one morning is found to be 14

(c) Test, at the 5% level of significance, whether there is evidence of a decrease in the rate of cyclists passing the point.
State your hypotheses clearly.

(3)

a) Let X be number of cyclists in a 10-minute interval travelling eastbound.
Let Y be number of cyclists in a 10-minute interval travelling westbound.

$X \sim Po(8)$

$Y \sim Po(3)$

$X+Y \sim Po(8+3)$

$X+Y \sim Po(11)$

The number of cyclists travelling eastbound is independent of the number of cyclists travelling westbound.

b) $\dfrac{P(X=11) \times P(Y=1) + P(X=12) \times P(X=0)}{P(X+Y=12)}$

$= 0.1204$ (4dp)

DO NOT WRITE IN THIS AREA

Question 4 continued

c) Let Z be number of cyclists in a 20-minute interval travelling eastbound.
Let W be number of cyclists in a 20-minute interval travelling westbound.

$Z+W \sim Po(16+6)$
$Z+W \sim Po(22)$

$Z \sim Po(8\times2)$
$W \sim Po(3\times2)$

$H_0: \mu = 22$
$H_1: \mu < 22$

$P(Z+W \leq 14) = 0.048$

$0.048 < 0.05$
so reject H_0 as there is sufficient evidence to suggest there is evidence that the rate of cyclists has decreased.

Question 4 continued

(Total for Question 4 is 8 marks)

TOTAL FOR FURTHER STATISTICS 1 IS 40 MARKS